ENRIQUE IGOA

FUNCTIONAL HARMONY: REVIEW AND UPDATE OF THE SYSTEM

(ENGLISH REVISED AND ENLARGED VERSION 2023)

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(English revised and enlarged version 2023)*

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Abstract

The harmonic-functional system is based on a conception of harmony and the chords of the tonality derived from the notion of differential hierarchy, fifth relationships and functional affinity, whose consequence is the recognition of three basic functions –tonic, dominant and subdominant– and of three other secondary or derivative functions, each with two variants, which complete the rest of useful chords as secondary tonic within a given tonality.

A revision of the harmonic-functional system must begin with the very introduction of the concept of 'function' in musical theory and, more specifically, in musical analysis. Secondly, it is necessary to recall and discuss succinctly the same foundation of harmony, according to several authors (Rameau, Riemann, Schönberg), although focusing only on those aspects of its theoretical tradition that concern the origin of the chord, its position within the system, its typology and relationships between chords.

Finally, the paper develops a complete view of the harmonic-functional system which provides an overview of all types of chords that are studied in any academic harmony treatise, but grouped according to their function in harmonic discourse, that is, summarized under the cover of the three basic functions and their derived functions, in an organic unit that immediately gives each chord its role in speech and thus its harmonic function. In this section we will review the definition of basic functions and secondary functions, the numbering of chord sounds, movement of voices, cadential functions (dominant, dominant of the dominant and subdominant), tonicization and modulation, to end with extensions of the system that allow to use it even in the limits of the tonal system.

Key words

Harmony, function, functional harmony, chord, musical analysis, Rameau, Riemann, Schönberg, Diether de la Motte

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Introduction

The harmonic-functional system is based on a conception of the harmony and chords of the tonality derived from the notion of differential hierarchy, of fifth relationships and functional affinity, whose consequence is the recognition of three basic functions – tonic, dominant and subdominant – and of three other secondary or derivative functions, each with two variants, which complete the rest of useful chords as secondary tonic within a given key.

A revision of the harmonic-functional system must begin with the very introduction of the concept of function in musical theory and, more specifically, in musical analysis. It is a term usually absent from musical dictionaries, although it is possible to find entries such as «functional harmony» in the Harvard Dictionary of Music, for example (Randel 1997: 463), as well as in the title of some important monographs, among them the own founding text of Hugo Riemann.[1] Nor is there a place for this concept in the general programming of those theoretical subjects where it could be relevant (harmony, musical forms, analysis, musical language, etc.), except for some exceptions. Therefore, it seems necessary a brief initial tour through the origin and application of this concept in the field of theory and analysis, to then focus on its use in the harmonic-functional system. This is the objective of Section 1.

Second, it is necessary to remember and discuss succinctly the same foundation of harmony, according to various authors, although focusing only on those aspects of its theoretical tradition that concern the origin of the chord, its position within the system, its typology, and the basic relations between chords.[2] These issues will be discussed in Section 2.

Finally, the paper develops a complete vision of the harmonic-functional system which, as will be seen, provides an overview of all the types of chords that are studied in any academic treatise of harmony, but grouped according to their function in harmonic discourse. In this way, what in these texts seems to be, sometimes, an extensive and sometimes confusing list of chords without a clear relationship between them, here it is summarized –under the protection of the three basic functions and their derived functions– in an organic unity that immediately gives each chord its role in the discourse and with it its harmonic function. In this space we will review the definition of basic functions and secondary functions, the numbering of the chord sounds, movement of the voices, cadential functions (dominant, dominant of the dominant and subdominant), tonicization and modulation, to finish with the extensions of the system that allow to use it even in the limits of the tonal system. All of this will be developed in Section 3.

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[1] Of the titles cited in the final bibliography, in this regard, stand out the text of Hugo Riemann (*Vereinfachte Harmonielehre oder die Lehre von den tonalen Funktionen der Akkorde*), that of Hugo Distler (*Funktionelle Harmonielehre*) and that of Arnold Schönberg (*Structural Function of Harmony*). And they endorse this presence classic books of music theory as the one published by William Caplin in 1998 (*Classical Form: A Theory of Formal Functions for the Instrumental Music of Haydn, Mozart, and Beethoven*), or his paper of 2005 entitled "On the Relation of Musical Topoi to Formal Function."

[2] There are papers and books that extensively develop everything related to the historical origin of harmony, the basis of the tonal system, harmony as a discipline in academic studies, etc. Virtually all texts cited in the final bibliography address one or more of these aspects. A panoramic vision is offered by the "Harmony" entry by Carl Dahlhaus et al. in the New Grove Dictionary.

1. Function definition

The term 'function' was incorporated into music theory and analysis many years ago, as a means to define and point out the role of parameters that converge in musical discourse. That is why it is one of the most important concepts in music analysis, with implications in all the disciplines that come together in this field. Starting from the different meanings offered by the Real Academia Española (Royal Spanish Academy) in its *Diccionario de la Lengua Española* (Dictionary of the Spanish Language) for the term function, it is possible to extrapolate several definitions for its application in the field of music theory.

From the generic definition[3] it is deduced that function is the capacity of action of any musical element, be it a sound or pitch, a motive, a chord or succession of chords, a rhythmic figure, a texture, a timbre combination, etc. The actions that a musical element can carry out, alone or in combination with other elements, are multiple: introduction, beginning, exposition, development, recapitulation, intensification, reduction, extension and, of course, cadential processes of varying degrees of importance, as well as articulation of discourse at various levels.

Of the three linguistic definitions (see Notes 2, 4 and 5) three variants are obtained. According to the first,[4] function is the role played by an element in the structure of musical discourse. While the capacity of action refers to the theoretical or latent possibilities of action in each element, the role of that element in the musical structure refers to a concrete realization, in which the former can fulfill one of its most usual missions, or else an infrequent or even unexpected action. Thus, for example, one thing is the usual task that performs the $\mathbf{D^7}$ ($\mathbf{V^7}$) chord within the classical key (that is, its conventional implication, which is to announce and precede a major or minor tonic chord), and something else are its frequent surprises or unexpected resolutions found in the repertoire of all time.[5]

From the second linguistic definition,[6] function is the relationship between the various elements of a musical structure, which in analysis can be understood in two ways. On the one hand, it may refer to the different functions of each element: melodic function, harmonic function, rhythmic function, etc., without forgetting that the elements can also assume a mission that is not their own, although in some cases it is so obvious like her. For example, in a 4-voice choral, harmony also has a relevant melodic function in all voices, in the sense that it is not a mere filling, as it happens in other textures. In many instrumental works the arpeggio – at least in name a succession of notes – has also or even essentially a clear harmonic function. The texture or the rhythm

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[3] Action aptitude of living beings, machines, or instruments (RAE [Real Academia Española], Diccionario: 1004).

[4] In the grammatical structure of the sentence, the role that plays a phonic, morphological, lexical or syntactic element (RAE: 1004).

[5] It could hardly be considered usual the use of the aforementioned chord that Moussorgsky proposes in the impressive Coronation scene from *Boris Godunov* (Prologue, Scene 2), in which the $\mathbf{D^7}$ of D flat alternates (during 42 bars!) with the $\mathbf{D^7}$ of G, its farthest key in the circle of fifths, without reaching either of its two objectives. The apparent absence of tonal centrality is mitigated, however, when it is observed that there are two sounds common to both chords, the C and the F sharp or G flat, and that both chords can also be interpreted as different forms of the $\mathbf{^D D}$ ($\mathbf{^V V}$) in C, the key in which they finally resolve some time later.

[6] Relationship that the elements of a grammatical structure maintain with each other.

have in some works of the last two centuries a motivic function, as it happens with the chord that opens Stravinsky's *Symphony of Psalms*, or with the 'Invention on a rhythm' used for 3rd scene of Act III in Berg's *Wozzeck*, respectively. Meanwhile, harmony often assumes a rhythmic function, and also at various levels, as it is known (rhythm of chords, rhythm of modulation, rhythm of tonality, etc.).

Second, within each element or parameter there is a functional relationship derived from a hierarchy that assigns a greater or lesser weight to each component or participant within the structure. Thus, they differentiate and relate the *finalis*, the *tenor*, the initials and the middle notes within an ecclesiastical mode, or the tonic, the dominant, the subdominant and its relative chords in the tonal system, or the different polarities in a non-tonal work, etc.

According to the third linguistic definition,[7] function would be the ability of the music and of each musical element to represent reality, transmit content and express themselves or other people's feelings, which leads us to the slippery theme of expression in music, about which so much has been written and on which the most opposing opinions have been expressed, and whose slightest discussion go far beyond the scope of this article.[8]

Finally, we focus on the mathematical meaning of the term,[9] which offers also interesting musical extrapolations. According to this definition, function is a rule between two sets which assigns to each element of the first an element of the second. This means musically so simple facts as the transposition of pitch, by means of which the sounds included in the scale which displays the sounds of a mode, the set of chords belonging to a particular key or a concrete pitch-class set in atonal music could all match another similar set, once established by which every sound or chord from the initial sets is raised or lowered according to a constant interval.

Of all these meanings we will retain here that which relates the concept of function with the chords, to serve as a foundation for functional harmony. But first it is necessary to make a brief incursion in the basis of harmony and the origin of the chords, to complete this preamble with the essential aspects of the other part of the system.

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[7] Each of the language's abilities to represent reality, express the speaker's feelings or incite the listener's performance (RAE: 1004).

[8] Without any intention of delving into such a thorny subject, I propose here only some clues for the most curious. Assuming that the music expresses something, transmits a message, has the ability to communicate or to expose some content, as a language would do in the broadest sense of the term, or to generate a specific emotional state in the listener, the truth is that any element or parameter can be the main carrier of such a message. In addition to remembering that there has been a whole system –effective for almost three centuries– based on these assumptions (musical rhetoric and its rhetorical figures, to which the theory of topics could be added), they may serve as examples of this assumption the following passages (which I prefer not to adjective, not to condition the perception of the listener): the melody of *Lacrymosa* in the *Requiem KV 626* of Mozart or that in *Adagietto* of *Symphony no. 5* of Mahler; the chords supported by the harp in *Adagio* of *Symphony no. 8* of Bruckner, the chords that open the symphonic poem *Also sprach Zarathustra* of R. Strauss or the parallel chords at the beginning of *Nuages* in *Nocturnes* of Debussy; the rhythmic figure that underlines the Death of Siegfried in *Acto III* of *Götterdämmerung* of Wagner, or the one that opens the No. 1: Mars, the Bringer of War of *The Planets* de Holst; the texture that closes the opera *Wozzeck* of Berg, or that of the 3rd piece from the *5 Stücke für Orchester op. 10* of Webern.

[9] In mathematics, a mathematical rule between two sets that assigns to each element of the first one another element of the second (RAE: 1004).

2. Fundamentals of harmonic theory: Rameau, Riemann and Schönberg

A definition of harmony –such as the one written by Carl Dahlhaus in the corresponding article in New Grove Dictionary– is, no doubt, a necessary starting point to open this Section:

> [Harmony is] the combining of notes simultaneously, to produce chords, and successively, to produce chord progressions. The term is used descriptively to denote notes and chords so combined, and also prescriptively to denote a system of structural principles governing their combination. In the latter sense, harmony has its own body of theoretical literature (Dahlhaus *et al.* 2001: 858).[10]

The same author recalls that the concept of harmony appears for the first time in the ancient Greek theory, although with a different sense from the later, whereas in the Middle Ages it refers to the combination of two notes – and in the Renaissance of three notes – that sound simultaneously, although it was Gioseffo Zarlino (1517-1590) the first theorist who based his concept of harmony on the idea of triads consisting of fifths and thirds. Shortly after he says: "The harmonic theory of recent times, which evolved gradually between 16th and the 18th centuries, is based on the idea that a chord – three or four notes sounded simultaneously – is to be taken as primary, as an indivisible unit" (Dahlhaus *et al.* 2001: 859).

Once the chord has been ratified as an entity, the next step is the elucidation of the concept of inversion, that is, the confirmation that the fundamental position of a chord (C-E-G) and the other arrangements of the chord (E-G-C and G-C-E) are "different manifestations of an identical harmony and that the bass note of the 5-3 form (C-E-G) must count as the fundamental note, the basis and center of reference of the other notes" (Dahlhaus *et al.* 2001, 859). This fact is crucial in the harmonic conception of Hugo Riemann, in which the numbering of the members of the chord does not change with his inversion, unlike what happens with the numbering derived from thoroughbass.

However, the notion of inversion, as well as many new developments, was to be consolidated much earlier, with the publication in 1722 of the *Traité de l'Harmonie reducite à ses principes naturels* by J.P. Rameau, a founding text of harmony as a theoretical tradition and as a discipline. The broad theoretical contribution of this composer has been studied extensively by authors such as Bernard (1980) and Lester (1992, 2002). The first one summarizes the way in which Rameau conceives the inversion in relation to the fundamental bass:

> Rameau was the first to postulate an equivalence of function between chords that were identical in pitch content but which had different bass notes. For example, C-E-G, E-G-C and G-C-E all had the same fundamental – that is, C. Accordingly, there could be found, separate and distinct from the *basso continuo*, a fundamental bass which was simply a succession of fundamental notes, one from each of the harmonies in the progression. The fundamental bass was not actually played, but in its abstract existence it served, for Rameau, as a representation of the origin of harmonies – for, in each chord, the fundamental note was the generator of the other notes, according to the principle of the sonorous body (Bernard 1980: 43-44).

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[10] All translations of texts in German or French are by the author of these pages.

Lester (1992, 2002), as well as Dahlhaus (2001) agree to emphasize that the notion of inversion of a chord in relation to its fundamental position can be already found in texts from theorists previous to Rameau, mentioning, among others, to such scholars as Otto Siegfried Harnisch (1608), Johannes Lippius (1610, 1612), Thomas Campion (1613) and Andreas Werckmeister (1702). Nevertheless, Lester underlines that Rameau accomplished in this area something that no one before him had done: "First, Rameau extended inversional thinking to all chord-types, not just triads [..] Second, and more important, he was the first to realize that the notion of chord roots could be the basis of a powerful explanations of harmonic progression" (Lester 1992: 100); whereas Dahlhaus writes: "What was decisively new in Rameau was not the theorem [of inversion] as such but its incorporation into a comprehensive theory of musical coherence, in which the conception of the chord as a unit, primary and indivisible, the concept of the rot note, the doctrine of the fundamental bass and the establishment of a hierarchy between the fundamental degrees were interdependent elements, complementing and modifying each other" (Dahlhaus *et al.* 2001: 860).[11] Lester has also emphasized that, thanks to Rameau, "the invertibility of chords – seventh chords as well as triads – thus became not just a curious trait of chords to be used as a mnemonic, but a path to understanding the behavior of harmony and voice leading" (Lester 2002: 760).

In the treatises that followed the *Traité*, Rameau introduces several novelties, as a direct consequence of scientific findings of the time. On the one hand, aware of the discoveries in the field of the physics of the sounds of Joseph Sauveur (1653-1716), he proposed as the generating principle of the chords the series of harmonics generated by the sound bodies in vibration (*corps sonores*). This is the case in the *Nouveau Système de Musique théorique* (1726). In this same treatise, as well as in the *Génération harmonique* (1737), he assumes a much more important role for the subdominant function, through the geometric progression 1:3:9, which he calls *triple proportion*, since each sound is the third harmonic of the previous one. If F were the first harmonic in a series, C would be the third and G the ninth. This allows him to propose a tonal organization based on a tonic surrounded by a higher fifth (the dominant one) and a lower fifth (the subdominant), where these last two chords include a third added above (the dominant) and below (the subdominant), to form in both cases seventh chords, which, placed along a line, shows a perfect symmetry around the tonic chord (Fig. 1). In this way is possible to reinforce the role of the fourth degree, which in its habitual use is built on the F and it is considered to have an added sixth (*sixte ajoutée*) when it resolves as irregular cadence (plagal cadence) in the tonic, while this sixth can be understood as the fundamental sound of the chord in its theoretical construction and in its resolution over the dominant. Ultimately, then, Rameau applies Newton's gravitational principle – whose theories were known in France in the 1730s – to the tonic chord, which 'attracts' both the dominant and the subdominant chords into its orbit to provide the sensation of tonality. With the primacy of these three functions he is also anticipating in almost two

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[11] Remarking the contribution of this author, Lester remembers that precisely because Rameau "built a large part of his new perspectives based on verifiable principles and musical facts, many of his ideas spread through the musical world very quickly [...]. His ideas on harmony transformed the counterpoint and the *basso continuo* [...] and for this reason the study of harmony was for a long time the center of the theory of tonal music [...]. Unlike the traditions of counterpoint and bass, which derive from the works of numerous writers, the theory of harmony achieved its prominence in the eighteenth century thanks to texts published by a single person over several decades (Lester 1992: 93-94).

centuries to the concept of basic functions that Hugo Riemann will formulate in his writings on harmony at the end of 19th and beginnings of 20th centuries.
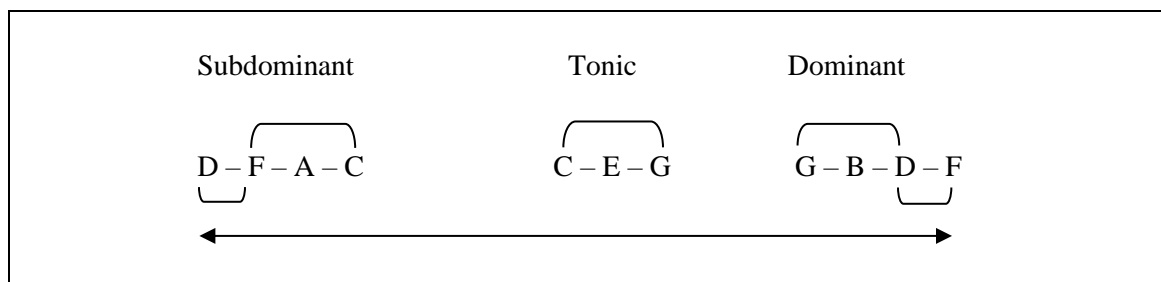

Fig. 1 Rameau's demonstration of the symmetric relation between dominant and subdominant with added third (according to Lester 2002: 769 and Rameau 1737)

From the basic harmonic intervals (fifths, major and minor thirds), Rameau generated their two basic types of chords, the triad and the seventh chord. All other chords can be derived from these (although in some cases with certain difficulties). On the other hand, he considered that the perfect movements of the bass were those of fifth, followed by those of third (harmonic intervals), being those of second considered like dissonant, for which reason he replaced in the fundamental bass certain sounds of the real progression with others that always allow the exclusive use of the first two types of interval, although in the thoroughbass the seconds were included (see Rameau 1722; Lester 1992 and 2002).

With these premises, he defined two types of cadence, the 'perfect' (authentic) cadence – which begins with a dominant with seventh, whose dissonance results in the need for resolution in a stable chord of tonic –, and the 'irregular' cadence (our plagal cadence), which begins with a subdominant with added sixth, dissonance that is also resolved in the tonic. Both were considered as the basis of the harmonic movement, which he considered as an interconnection of cadences, many of which were evaded by some local alteration. "With perfect, irregular and evaded cadences, Rameau tried to show how the fundamental bass proceeded primarily by fifths and thirds – the very intervals generated from the fundamental string. He thereby explored the recently developed sense of tonal directionality that differentiated the music of his time from that of earlier generations" (Lester 2002: 763).

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The chapter entitled by Lester "Changing Aspects of Harmonic Theory" in his book on *Compositional Theory in the Eighteenth Century* (Lester 1992: 193-230) shows the direct or indirect influence of Rameau's theories in the treatises written in the second half of 18th century. We will take from them those aspects centered in the new perspectives around the three basic functions and other forerunner contributions to the upcoming functional harmony before Riemann.

Johann Friedrich Daube (c. 1730-1797) wrote his *Thoroughbass in Three Chords* (1756) "based on the *tonic chord* (*Grundtonaccord*) or triad on scale-step 1, the fourth chord *(4ten Accord)* or added-sixth chord on scale-step 4, and the *fifth chord* (*5ten Accord*) or seventh chord on scale-step 5. According to Daube, all the harmonies in music arise from these three chords, their inversions (*Umwendungen*), anticipations and suspensions of notes of these chords, or imitation of these chords" (Lester 1992: 200). This brief writing was preceded by another thoroughbass manual by Georg Joachim Joseph Hahn (c. 1690-after 1769) that anticipates Daube's approach: "Like Daube, Hahn cites three principal chords (*Haupt-Accorden* or *Haupt-Klänge*) on 1, 4 and 5, whose original form or inversion (*Veränderung* or *Verwechselung*) ordinarily appears over each bass note. Hahn does not announce this harmonic approach as a novelty, but simply places it alongside traditional thoroughbass material" (Lester 1992: 202). Going back to Daube, it cannot be denied that even the notion of secondary dominant seems to be very familiar to him "as the standard explanation for changing key by introducing a new leading tone: «the harmony of the foreign *third chord* [the chord on the new scale-step 5, i.e., the secondary dominant] must always precede the appearance of the foreign *first chord* [the new tonic chord, i.e., the secondary tonic]»" (Lester 1992: 2013).

Concerning key changes and chromaticism, one of the best contributions is due to Friedrich Wilhelm Marpurg (1718-1795), who "notes that the «rule of unity» demands one key for a whole piece, while the «rule of variety» requires the presence of other keys. From this perspective, theorists needed to explain how separate keys relate to one another and how progressions can be led from one key to another" (Lester 1992: 214). The various proposals to explain this topic go from the basic one – "the closely related keys are those whose tonic triads exist in the original key" –, to more subtle ideas: "the keys closest to the tonic in circular chord progressions of rising or descending thirds proposed by Werckmeister (C-e-G-b-D… and C-a-F-d…); […] the keys closest to the tonic on Heinichen's musical circle and on Mattheson's «improved» circle of 1735. All these schemes incorporate both major and minor keys into a single progression […] and various writers began to include keys related by major-minor mixture in the orbit of diatonically related keys" (Lester 1992: 215).

The change of key considered as a local displacement or as more permanent tonal movement has been discussed in one way or another by many 18th century theorists. Lester has recollected the main options:

> […] thoroughbass theorists in occasional remarks and harmonic theorists from Rameau onward suggest that brief instances of chromaticism can also be viewed as a temporary digression within the prevailing key. Georg Andreas Sorge (1703-1778) notes that sharps can occasionally appear without an actual change to another key. Johann Philipp Kirnberger (1721-1783) and Heinrich Christoph Koch (1749-1816) came yet closer to specifying the difference between such local progressions and an actual change of key. But [..] until terms such as secondary dominant, applied dominant, borrowed chord, or auxiliary dominant, became common, most such discussions remained isolated and did not become universal (Lester 1992: 217)

[…] Kirnberger also explains more clearly than earlier theorists the distinction between what we call nowadays local tonicizations and larger changes of key, as well as the difference between passing through keys and fully establishing a new key, which requires a cadence to confirm the new tonic (Lester 1992: 244).

Joseph Riepel (1709-1782), one of the most significant theorists of the second half of 18th century, wrote his monumental *Anfangsgründe zur musicalischen Setzkunst* between 1752 and 1768. Apart from the principal concern of Riepel, which is melody and phrasing, there are in its pages some remarks around harmony, where it is evident a contradiction between his "antiquated manner of dealing with harmony" and its application of "many aspects of Rameauian harmony". Here comes to the fore once more the existence of just "three basic chords: those on scale-steps 1, 4 and 5. All other chords are built on *subsidiary* or *supplementary tones* or *intermediate bass notes* (*Neben-* or *Ausfüllungstöne* or *Mittelbassnoten*)" (Lester 1992: 270).

Finally, one of the most influential scholars of the century (going beyond to the next century), H.C. Koch, assumes the same premise, inside his full and comprehensive approach to musical structure: "The triads on scale-steps 1, 4 and 5, which give rise to the key, are the essential (*wessentlich*) chords, while the triads on scale-steps 2, 3 and 6 of major key are incidental (*zufällig*) […] He describes innumerable harmonic usages via his notion of essential chords that enunciate a tonal area complemented by incidental chords that provide other avenues of harmonic motion, and via his notion of which chords can begin a phrase or section and how formal cadences provide a proper ending" (Lester 1992: 276, 284).

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There is no doubt that a significant part of Rameau's theories (along with those other contributions mentioned above) was exploited – with proper revisions and updates – by Hugo Riemann, a century and a half later, consciously or not. The fact is that today it is possible to appreciate an obvious connection in relation to one of the fundamental pillars of both formulations: the gravitation of the dominant and subdominant around the tonic in the first, and the establishment of the three basic functions in the second. Riemann set out the essential arguments of the system of harmonic functions in his text *Vereinfachte Harmonielehre oder die Lehre von den tonalen Funktionen der Akkorde* ("Simplified Treatise of Harmony or the teaching of the tonal functions of the chords"), published in 1893 and reissued in 1903, culmination of two decades of evolution and revision of its harmonic theories. Although some of the symbols used by the author have been modified by later scholars of the German area (Wilhelm Maler, Hermann Grabner, Hugo Distler, Diether de la Motte), much of their conceptual and terminological innovations remain fully in force, as we will see in Section 3. But previously, we have to remember the foundations of his system, which lies on the mentioned concept of function – visible already in the subtitle of his book, as his biographers recall: "borrowed from mathematics, the metaphor [the idea of function] registers a change in its harmonic theories from a concern with cadential successions to the adumbration of a quasi-algebraic musical logic based on functional relations between harmonies" (Hyer and Rehding 2001: 363).

Riemann begins by isolating as *Hauptklang* (main sonority or main chord) the tonic chord (**T**), which is the first basic function of tonality. From it he derives directly the chord built above the tonic with the higher sound of the tonic as fundamental, that is, the dominant one (**D**), and the chord built below with the fundamental of the tonic as fifth of the new chord, that is, the subdominant chord (**S**). Both chords complete with the tonic the cast of the three basic functions, the essence of harmony. He himself expresses it in a graph (Fig. 2) that resembles the one shown above to explain the symmetric relationship in Rameau (Fig. 1), and summarizes it in this sentence: "There are only three tonal functions in Harmony (significant functions within tonality), namely, the Tonic, Dominant and Subdominant. In the change[12] of these functions lies the essence of modulation" (Riemann 1893: 9).[13] Dahlhaus puts the basic functions in relation to the cadence to explain the tonality: "In contrast to the theory of degrees [*Stufen*], the functional theory of Riemann begins with the cadence **T-S-D-T** in order to establish the tonality, and deduces the scale by analyzing the three main chords (C-E-G, F-A-C, G-B-D = C-D-E-F-G-A-B-C)" (Dahlhaus et al., 2001: 862).
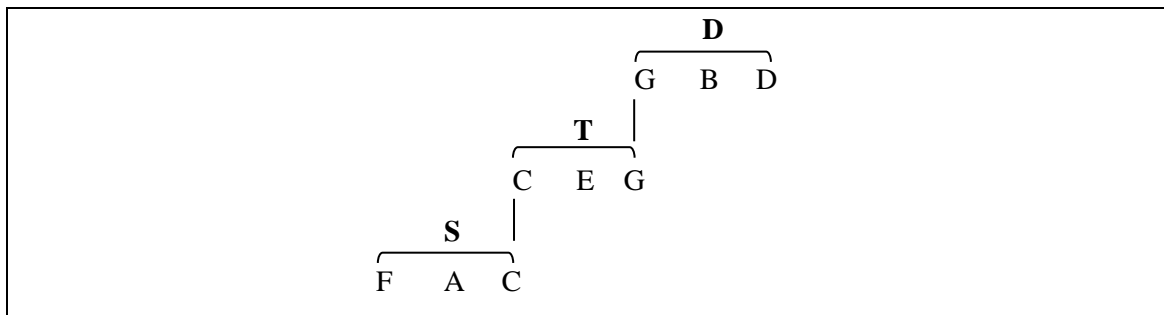

Fig. 2 Basic functions according to Riemann. Coupling of tonic function with dominant and subdominant (Riemann 1893: 8)

The derivation of the basic functions in minor mode is equivalent to that of the major mode, but the three resulting chords (**t**, **d**, and **s**) are minor, since they are taken from the natural scale. It is appropriate to remember, precisely, the relationship that establishes Riemann between the major and minor chords, or, put another way, the explanation of the origin of both chords: "From the combination of a sound with its major third and its upper fifth originates the higher resonance (major chord), and from its combination with its lower major third and its lower fifth originates the lower resonance (minor chord)" (Riemann 1893: 10). This gave validity to the theory of Rameau (strings of F and A flat that vibrate by sympathy below the C) and to the concept of lower resonance, an idea inherited from Arthur von Oetttingen (1836-1920), who formulated in 1866 the theory of «harmonic dualism», by which the minor triad is generated below a fundamental tone, as opposed to the major triad, which is generated above that note.

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[12] He refers to the exchange that occurs when passing from one tonality to another – either in a tonicization or in a modulation – which necessarily implies that the three basic functions move also from one fundamental to another according to the implied tonal transposition.
[13] All the translations from German to English are mine.

There are three possible transformations from the basic functions, which Riemann called «variant, parallel and change of leading tone», in which there is always a common interval. The «variant» is nothing more than the relation between major and minor chords that share the same fifth, that is, the two keys with the same name and opposite modes (e.g., C major and C minor). The relation of «parallel» is found in the two chords that share the same major thirrd (e.g., C major and A minor), that is, what is usually known as relative chords.[14] The «change of leading tone» (*Leittonwechselklang*) is the relationship between two chords that share the same minor third (e.g., C major and E minor), a relation that lacks theoretical tradition.[15] In the case of parallel chords, the existence of a common interval has two important consequences: on the one hand, the chord derived as a parallel is considered by Riemann as an apparent consonance (*Scheinkonsonanz*), and, on the other hand, the derived chords can stand for the original functions (*Klangvertretung*), something which enables the parallel chords to assume the function of the basic chord in many contexts.[16] According to the author, "the introduction of parallel chords supposes a significant enrichment of the means of construction of the cadence and a more subtle breakdown of them" (Riemann 1893: 80). The harmonic-functional system supposes, therefore, a criticism to the numerical ciphering of the seven degrees (I, II, III, etc.) and to its acceptance of a presumed equality between them; the construction of chords that he proposes – based in the harmonic sense (the function) of these degrees – makes evident the essential difference between basic chords and derived chords, as a consequence of their position in the tonal hierarchy (Riemann 1893: 132). It should be stressed that the roman-numeral ciphering is merely a numbering of chords providing no essential differences between them; but functional harmony, on the contrary, 'explains' chords and its role in musical discourse by assigning them to one of the three basic functions, either as the basic chord or as one of the two derivatives (parallel or counter-chord) of these basic chords. There are many other advantages of functional harmony – both as a means to write harmonic exercises and as a tool for harmonic analysis – that will be described and developed in Section 3.

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Structural Functions of Harmony, Arnold Schönberg's second contribution to harmonic theory after his *Harmonielehre* of 1911, was published posthumously in 1954. The title of the book makes clear the intention to unite the concept of function with the chord as active subject within the musical structure. For this Schönberg establishes the idea of 'underlying principal key' or 'monotonality', associated with a unitary hearing perception. The various secondary tonics – object of possible local modulations – that can be obtained from the central tone as relative chords or by chromatic changes, are called 'regions', in the sense of small spaces of the supposed large territory that is the main key, and whose generally brief presence does not endanger the sovereignty of this

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[14] Riemann introduced the notation **Tp**, **Dp** and **Sp** for the parallel (relative) chords of the basic chords in major mode. The lowercase letter **p** indicates that the resulting chord is minor. However, for the equivalent in minor mode he used the **⁰Tp**, **⁰Dp** and **⁰Sp** symbols, which have been replaced in the current functional theory by the more logical **tP**, **dP** and **sP**, where the capital letter indicates that the parallel (relative) chords of these basic minor functions (**t**, **d**, **s**) are always a major chord (**P**).

[15] The evolution of the Riemannian system led to replace this denomination by «counter-chord» or «counter-parallel» (*gegenakkord*), as we will see in Section 3.

[16] Riemann gives as an example the conduction in a deceptive cadence from **D** to **Tp**, where this chord is an "apparent consonance" with respect to the expected chord (the **T**) and at the same time represents the tonic function in this context (Riemann 1893: 79).

main key. It is also possible to find a visual analogy by associating the tonality with the white color (which is the sum of all the colors), so that the tonal regions would be moments of diverse coloration in the middle of that prevailing white color.

Schönberg exposes his harmonic principles and its extent in the unified system of the monotonality applying it to the analysis of all type of works composed between 18th and 20th century. In this way, what he himself previously considered as modulation (in his *Harmonielehre*) is now considered to be what the harmonic theory calls 'tonicization' (*Tonikalisierung*), that is, the momentary displacement to a harmonic region near or far within the same tonality. However, both Riemann[17] and Schönberg are aware that a harmonic movement towards a tonic that is established during a significant section of a work can (and should) be considered as a modulation, and the clearest case is the sonata form, its derivatives and analogous genres, with their contrasting tonal area in the second part of the exposition or in the contrasting sections.[18] In this case, there will be two (or more, in works of a certain complexity) the reference keys with respect to which the displacements to their internal regions are valued. Schönberg does not conceal the difficulty of analyzing certain musical fragments in which distant regions, mode changes, wandering chords, etc. are used, trying to put moreover this sum of secondary tonic in coherent relation with a single main tonality. We will return to this problem in the corresponding section.

The proximity or distance of the regions with respect to the central tonic is shown by Schönberg displayed in a triple line that shows the chords derived from the three basic functions and their progressive distance through chromatic alteration or change of mode (Fig. 3a). The terminology is based in the following symbols: major and minor tonic (T/t); dominant major and minor (D/v), major and minor subdominant (SD/sd); minor and major mediant, major and minor lowered mediant (m, M, ♭M, ♭m); minor and major submediant, lowered major and minor submediant (sm, SM, ♭SM, ♭sm); dominant of the major lowered mediant (♭MD) and fifth grade of the lowered minor mediant (mv); major and minor supertonic (S/T and dor [Dorian]); and the lowered second degree (Np, Neapolitan) (Schönberg 1954: 20).

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[17] Riemann already theorized about this process in his *Vereinfachte Harmonielehre*, and the terminology he established to indicate in harmonic analysis the presence of secondary chords (**D**, **S** or **ᴰD**) and secondary tonic remains fully valid in the current system, as can be seen in the *Harmonielehre* of Diether de la Motte.

[18] Schönberg clarifies it at the beginning of the study of the regions: "One should not speak of modulation unless a tonality has been abandoned definitively and for a considerable time, and another tonality has been established harmonically as well as thematically" (Schönberg 1954: 19).
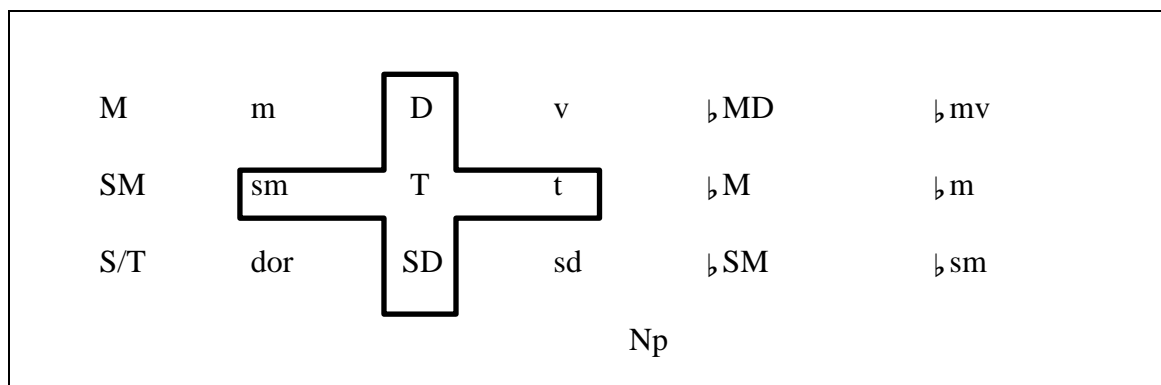
M	m	D	v	♭MD	♭mv
SM	sm	T	t	♭M	♭m
S/T	dor	SD	sd	♭SM	♭sm
			Np		

Fig. 3a Chart depicting the harmonic regions in a major mode according to Schönberg. Those included in the cross are considered as the closest to the tonic (Schönberg 1954: 20)
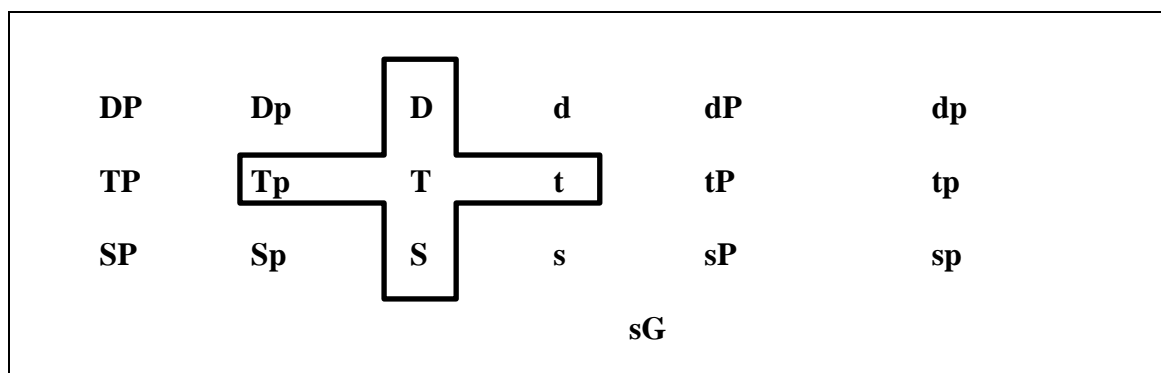

DP	**Dp**	**D**	**d**	**dP**	**dp**
TP	**Tp**	**T**	**t**	**tP**	**tp**
SP	**Sp**	**S**	**s**	**sP**	**sp**
			sG		

Fig. 3b Table showing the same harmonic regions in a major mode according to Schönberg using the revised terminology of the harmonic-functional system

Fig. 3b reproduces the same chart as Fig. 3a using the terminology of the revised harmonic-functional system presented below. It can be seen that the three basic functions of Riemann, together with one of the derivative chords (**Tp**, the sixth degree) are included in the group of the five harmonic regions considered as closest, to which Schönberg adds the minor tonic (**t**). However, it leaves the chords both on the third and the second degree outside the central cross (**Dp**, **Sp**). In any case, the origin of all near and far chords is very visible from the three basic functions, by changing the mode of the basic function itself and/or that of its derivative. In paragraph 3.8.4, dedicated to the levels of intermodal relationship, a similar formulation is proposed, including all the possible combinations of chords within the tonality.

The author proposes later a similar chart for the minor mode (Schönberg 1954: 30), and completes his essay with the study of the transformations of the chords, the vagrant [wandering, errant] chords (chords of diminished seventh, augmented fifth and sixth, which by means of the enharmonic principle may belong to various keys, as we will see in subsection 3.7), the major-minor interchange and a classification of the tonal relationships from the closest proximity to the maximum distance, another formulation of our levels of intermodal relation, as we will see.

3. Description and review of the harmonic-functional system[19]

The harmonic-functional system proposed by Hugo Riemann, later developed by Wilhelm Maler and Diether de la Motte, among others, can be summarized with a few principles which will be exposed next. These principles are not to be found, however, in the texts of these scholars; actually, they are the consequence of a process of merging and reviewing I have carried out from these theories, with the object of simplifying to the utmost the harmonic concepts inside the field of analysis. This kind of harmonic-functional analysis is applicable to all music composed in the reign of bimodal tonality, i.e., from the central and late Baroque to the end of 19th century, although it may be found in some works from 20th century based in the extended tonality (see Schönberg 1954: 76-113).

3.1. Basic functions

Tonality is a system of hierarchies, in which the functional relationships revolve around a tonic chord and the two chords that accompany it at a distance of upper and lower fifth, the dominant and the subdominant. There are, therefore, three basic or primary functions in tonality: the tonic, the dominant and the subdominant, whose temporal presence in the most classical tonal works is much greater than that of the rest of the functions.[20] All three can be represented by major or minor chords. The symbols that represent them are letters that will be capital letters in the first case and lowercase letters in the second: **T/t**, **D/d**, **S/s**.[21] Among them there is a common sound if they are arranged in the order **S – T – D** or **s – t – d**. In the analysis, the presence of a single letter is, therefore, a sign that we are dealing with basic or primary functions.

3.2. Secondary functions

There are two types of relation between chords that serve to obtain the remaining degrees of tonality, i.e., the secondary or derivative functions. The correspondence of relatives is established between two chords whose fundamentals are a minor third apart and, at the same time, they share two notes located a major third apart, as Riemann has already observed. If these chords were tonalities, it is easy to confirm that they would share the same key signature, that is, the same number of accidentals. Since the German

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[19] The following pages begin with a review of the terminology proposed in his *Harmonielehre* by Diether de la Motte (1976/1989), which in turn derives from Hugo Riemann (*Vereinfachte Harmonielehre*) and Wilhelm Maler (*Beitrag zur durmolltonalen Harmonielehre*). In this review I have simplified some symbols and added others in order to clarify certain tonicizations, as will be explained later. The rest of the section is devoted to the cadence, the chords involved in it and its variants; to tonicization and modulation; and to the extensions of the system, including the levels of intermodal relation.

[20] Classical works, in the sense I use this term here, are those written during the second half of the 18th century by Haydn and Mozart, as well as by many other authors of this same period who cultivated the classical style in its most paradigmatic version, beyond adjectives such as 'gallant style' or of trends like the *Sturm und Drang*. The temporary presence of these functions can be perfectly measured, in order to interpret the data obtained, using the techniques of statistical analysis (see Igoa 1986, 1989, 1998, 1999).

[21] While the **T/t** and the **S/s** maintain their functional identity in both modes, the functional dominant can only be represented by the chord in major mode (**D**), since the **d** (minor V) is the chord as it is found in the minor scales, and since it is a minor chord it does not have the capacity to serve as a functional dominant in the cadential passages. It is, therefore, just a natural or diatonic dominant, usable in any context except the cadential one.

term for relative is *parallel* (parallel), in order to keep the original terminology we will use here this naming and the corresponding letter (**P/p**).[22] It is evident that, coming from major chords, the resulting relative chords are always minor chords and their fundamentals are located a minor third below, and coming from minor chords the resulting relative chords are major and their fundamentals are located a minor third above. In minor mode the basic functions are the natural chords built over the 1st, 4th and 5th scale-step according to its key signature, being then minor chords, including the dominant one, which here is to be considered as a natural or diatonic dominant.[23] The mode of resultant chord may be seen in the second letter. The most important consequence deduced from this relation is that every basic chord may be represented or replaced by its relative without losing its functional capacity, a concept which allows the right understanding of lots of passages which would be not convincingly explained with another type of ciphering, as we will see frequently in these pages.[24] In analysis, the reading of two letters (parallel or relative of the tonic, of the dominant or of the subdominant) warns about the presence of derivative or secondary chords (see Fig. 4).

The other secondary or derivative function is obtained from the opposite relation as the one explained above, always keeping two common sounds (here a minor third), although in this case the fundamentals are located a major third apart. It has no tradition in English musical theory, and its name in English – «counter-chord» – is the literal translation of German term *Gegenakkord*.[25] Here too the counter-chords derived from major chords become minor chords and their fundamentals are located a major third above, and the counter-chords derived from minor chords become major chords and their fundamentals are located a major third below. The counter-chords and their basic chords have, as we have seen, two common sounds, although in this case there is not a common key signature, but a difference of one accidental above or below. The mode of the resulting chord is obtained here also from the second letter of the ciphering, and the concept of functional affinity already explained for relatives is equally applicable, although with certain reservations that will come to light in next pages and which will be discussed in paragraph 3.8.1 (see Fig. 4).

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[22] James Hepokoski and Warren Darcy (*Elements of Sonata Theory: Norms, Types and Deformations in the Late-Eighteenth-Century Sonata*) use the term parallel referring to a major key and to the same key in minor mode, which is a very inadequate and confusing naming. I will maintain the term parallel for relative chords (C major and A minor, or D minor and F major), as it has been explained, and I will refer to a major tonic in relation with the same tonic in minor mode (C major and C minor), or to a minor tonic in relation with the same tonic in major mode (A minor and A major) as its opposite tonic.

[23] A very different thing, as we will see in Note 21, is the use of the functional dominant, i.e., a chord with major third (and sometimes also of the major subdominant) in authentic cadences in minor mode, in which case capital letters will be displayed (**D**, **S**). These chords will be treated as a borrowing from major mode or an interchange with it, in order to allow the presence of a leading tone between the third of the **D** and the fundamental note of the minor tonic (**t**). This connection is a necessary condition for the tonal cadences, if modal sonorities are to be avoided. Nevertheless, the old church modes (in particular the Aeolian mode, and also partially the Dorian and Phrygian modes) are present up to a point in minor scales, a fact generating a very interesting modal-tonal stress which has been exploited by composers in every time.

[24] A quick example of this question may be seen, as Riemann already observed (see Note 12), in the deceptive cadence, whose traditional analysis (V – VI) displays just two numbers, whereas with the functional analysis (**D – Tp**) it is very clear the shifting of an authentic cadence which in the last moment replaces the tonic as a goal with a chord derived from that tonic which, in this sense, keeps a functional affinity with it (something physically evident in the two sounds they share).

[25] This chord is the *Leittonwechselklang* of Riemann (see Note 11). Grabner calls it «counter-relative», and he also uses the term «counter-parallel».
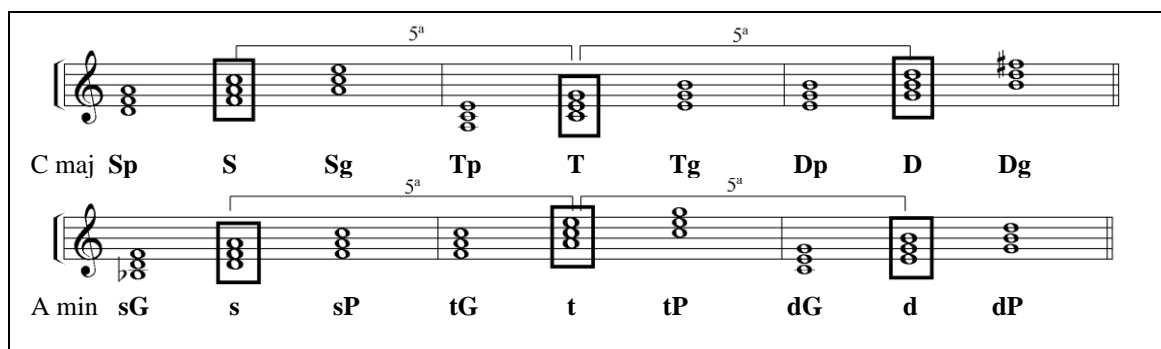
Fig. 4 Basic chords, parallel chords and counter-chords in a major and a minor tonality

Fig. 4 shows an apparent incoherence in the system which needs to be resolved. As it is evident, the three relative chords of the basic functions (**Tp**, **Sp**, **Dp** in major mode; **tP**, **sP**, **dP** in minor mode) are always chords belonging to the tonality (diatonic degrees of to the key), but two of the counter-chords have accidentals outside the key signature. None of them are derivatives of the tonic function **T**/**t**, coming instead from a major dominant (**D**) and from a minor subdominant (**s**), and being the relation of counter-chord the cause of this anomaly, it seems to indicate a lesser closeness of this secondary function with respect to the basic function than that of relative function. Nevertheless, the principle governing the construction of relatives and counter-chords requires that the resultant chords must be stable, that is, they must have a perfect fifth, and this is the one of the reasons for the accidentals.[26]

The counter-chord in major mode (**Dg**) is a tonal region scarcely used, unlike the rest of secondary functions, whose presence as an object for tonicization is customary – to a greater or lesser extent – in any type of tonal music.[27] At all events, it is relevant that the accidental used (F sharp) is the most frequent – statistically – en C major (an accidental also used in function **DD**, as we will see below), and this justifies to a great extent its presence.

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[26] The diatonic chord on the VII degree in C major or on the II degree in A minor (B-D-F in both cases, without accidentals), will be explained below, related with dominant and subdominant functions.

[27] Two famous examples which display the **Dg** as a secondary or contrast tonic are the Bruckner's *The Deum* (whose imposing onset in C major is soon darkened by a piercing B minor) and the tone poem by R. Strauss *Also sprach Zarathustra*, in which this opposition of tonalities has a symbolic purpose (C major for the nature, B minor for mankind). The harmonic progression in Prelude's (mm. 37-44) of *Suite Nr. 3* BWV 1008 in C major of J.S. Bach displays a model based in a **T** going down to a **Tp**, and this pair of chords goes up, so that the first repetition uses **Sp** and **Dg**. The reason to keep always the perfect fifth in all chords is, of course, of a technical nature.

The counter-chord of subdominant in minor mode (**sG**), however, has been a very favored chord by composers from 17th century, not in its root position, but in what today we would call the first inversion, that is, the famous Neapolitan sixth.[28] Also here the accidental used (B flat) is the most frequent – in the flat direction – in A minor.[29]

3.3. Numbering of sounds in chord

The functional-harmonic system assigns the terms and numbers 'fundamental' or first, third and fifth for the three sounds that make up the chord in its root position, like the traditional roman numeral ciphering. The main difference lies in the consideration of inversions. The traditional ciphering implies a new numbering of the sounds in the first inversion: counting from bass (now the third of the chord), the fifth becomes a third and the fundamental becomes a sixth. In the second inversion (built over the fifth of the chord) the fundamental becomes a fourth and third becomes a sixth. All these changes of numbering produce an unnecessary confusion in the construction of chords.[30]

The harmonic-functional system, on the contrary, keeps the numbering of the chord sounds in all the positions of the chord, thus applying the concept of function also to the role of each member of the chord, a function that does not change with inversion. That is, the fundamental (first or octave) is always the fundamental, even though it is not in the bass (and even when it is omitted), and the same happens with the third and the fifth.[31] Thus, in the first inversion the third is in the bass, and above are the fifth and the octave of the fundamental (and perhaps the duplicated third). In the second inversion the fifth is in the bass, and above are the octave and the third (and perhaps the duplicated fifth). All this is reflected in the harmonic analysis with a 3 or a 5 at the bottom of the letter that represents the chord in question, thus avoiding the combinations of numbers that are used in the traditional ciphering of inversions (Fig. 5).

C major: **T**   **T₃** **T₅**   **T⁴**   ³   **T₄**   ₃   **D₃⁹**   ⁸   **D⁴** ⁶₄ ⁵₃

Fig. 5 Chord root position and inversions. Movement of voices

[28] To be exact, the Neapolitan sixth is not exclusive of Naples (it was used throughout Italy in the expressive context of opera and oratorio), nor is it an inversion, as it might seem today, of a supposed chord (just the **sG**) which – curiously enough – was almost never used in its root position (at least until 19th century). The origin of this chord will be clear below, in the light of the principle governing the construction of subdominant function.

[29] For statistical checking of the presence of the twelve tones in different works along musical history I refer to my articles around the «Degree of Chromaticism» and "Statistical Analysis» (Igoa 1986, 1989, 1998, 1999), where there are several examples to corroborate it.

[30] This is an evident heritage of Baroque thoroughbass, whose ciphering is very effective to realize chords over a given bass in the context of works with this *continuo* writing. Nevertheless, its extrapolation to the teaching of Harmony (both in the realm of the writing of exercises as in harmonic analysis) has been one of the most flagrant mistakes of the harmonic theory.

[31] This issue was already explained by Riemann with many examples in the chapter devoted to the resolution of dissonances associated with the use of suspensions and *appoggiaturas*, although his numbering is valid also for analysing the passing and neighbour tones, anticipations and other melodic embellishments, as he proves with examples of double and triple suspensions (Riemann 1893: 119-123).

3.4. Movement of voices

Starting from the concept of invariable function just explained, it is easy to represent the movement of the voices originated in melodic motion.[32] It is just necessary to keep in every moment – in the root position as well as in inversions – the fixed numbering of each member of the chord, even when the fundamental is omitted (the remaining components of the sound are still numbered in its original position). In many cases the ciphering used here is the same as the employed in traditional scale-degree notation (Fig. 5), as it is the case in $\mathbf{T^{4\,3}}$, or in the double sixth and fourth *appoggiatura* in the authentic cadence ($\mathbf{D}^{\overset{6\ 5}{4\ \ 3}}$).[33] Nevertheless, is in the *appoggiatura* $\mathbf{T_{4\,3}}$ (where the movement of the voices is the same as in $\mathbf{T^{4\ 3}}$, but located in the bass) or in the *appoggiatura* $\mathbf{D}_3^{9\,8}$ (which is considered from the real fundamental of the chord, a G, and not from the occasional bass), where it becomes evident the coherence implied in the maintenance of a fixed numbering for the members of the chord.

3.5. The cadential functions

The authentic cadence is melodic, harmonic and rhythmic process used to confirm periodically the main key in a work, to reach a secondary tonic or to deviate to any of the possible tonicizations which take place along it (Fig. 6). The final goal of an authentic cadence is a chord with tonic function in major or minor mode (**T**/**t**). The function which announces and prepares the arrival to this tonic is a dominant chord, always in major mode (**D**). The usual chords which, in turn, precede the **D** are: a chord with subdominant function (**S**/**s**) and/or a chord with dominant of dominant function (**^{D}D**). In many cases both chords are used, almost always in the order $\mathbf{S/s} \rightarrow {}^\mathbf{D}\mathbf{D} \rightarrow \mathbf{D}$, because in this way the implicit chromaticism $\hat{4}$ - $\hat{4}\sharp$ - $\hat{5}$ will be realized according to the usual melodic rule. Of course, there are also examples in which the order changes to $^\mathbf{D}\mathbf{D} \rightarrow \mathbf{S/s} \rightarrow \mathbf{D}$, and this provokes an irregular melodic motion ($\hat{4}\sharp$ - $\hat{4}\natural$ - $\hat{5}$).
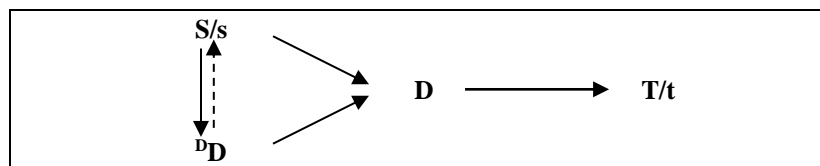

Fig. 6 The authentic cadence. Diagram of possibilities

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[32] I refer to the so named «tones foreign to harmony» (*Harmoniefremde Töne*), whose presence in a treatise of Harmony provoked an ironical astonishment to Schönberg (1922/1983: 309).

[33] Riemann warned: "The chord known usually as the antepenultimate chord in an authentic cadence, a tonic chord with the fifth in the bass (**T₅**), which is followed by a dominant in the road to the final tonic, is not really a tonic, but a dissonant form of the dominant, the so known 'sixth and fourth chord', i.e., a dominant with two foreign tones [a double *appoggiatura*] which move to the fifth and the third of the chord" (Riemann 1893: 22-23). Nevertheless, later texts like those of Schönberg himself (1954: 12-14) and Walter Piston (1941/1959: 117 and ff.) still analyze wrongly this cadence as a $\mathrm{I}^{\overset{6}{4}}$ followed by a V, although in both cases they consider the existence of a double appoggiatura, and, consequently, the existence of a single chord in two phases.

Caplin corroborates this diagram establishing three fundamental functions to clarify chords and harmonies in a tonality: the tonic function, the dominant function and the predominant functions (1998: 23-24), which include our **S/s** and **ᴰD**. These two last functions, however, show some dispersion concerning its designation, due to the understanding of chords following the traditional scale-degree ciphering, a problem which harmonic-functional system resolves when considering all these chords just as variants of the **S/s** or **ᴰD** functions.[34]

The tonic function **T/t** is always a triad chord which may just change its mode. The **S/s**, on the other hand, may be represented by few chords which can be triads or seventh chords, as we will see below. The other cadential functions display an evident parallelism between them, and this is the reason for studying them together. They are the **D** and the **ᴰD** functions, which include many variants of very different sign.[35]

3.5.1. The dominant function

At this point we will deal with the two dominant functions which are used in the traditional tonality, the dominant function and the dominant of the dominant function, since their affinity makes advisable to consider them as parallel functions, built however over different scale-degrees and with different temporal locations in cadential process (the latter usually precede the former). We will deal in this paragraph with the dominant function, and the dominant of dominant function will be treated in paragraph 3.5.2.

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The best way to understand the dominant function (**D**) is to consider it as represented by a great family of chords, a series of variants originated in the combinations of the sounds of a supposed 5-note chord built over the fundamental (the 5th degree of the scale), its third, its fifth, its seventh and its ninth (see Fig. 7). The essential conditions for the function to keep its identity are:

- The fundamental may be present or may be omitted. In any case, the numbering of the rest of sounds remains unchanged. The absence of the fundamental is indicated with a diagonal line crossing the capital letter (**Ð**).
- The third is always major (since it is a functional dominant), and must be always present.

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[34] I refer to chords labeled by Caplin as II^6, IV, II^6_5, IV, IV^6 and so on, all of which must be considered as different forms of the subdominant function. Worst is the case with the extensive group of chords labeled as $(V) \rightarrow V$, $(VII^7) \rightarrow V$, A^6 (Ger., It, Fr.), and so on, since there is a mix of roman numerals (secondary V and VII of V) with capital letters (A^6, that is, augmented sixth, with its German, Italian or French variants) to denote something which in harmonic-functional analysis is just one function, the dominant of the dominant, with many variants according to the presence or absence of its fundamental, the type of fifth employed (perfect or diminished), the possible use of its 9th, etc., as we will see in paragraph 3.5.2.

[35] Dahlhaus explains that, in the cadential understanding of Riemann, the individual chords may be omitted or altered chromatically without losing the cadential sense (Dahlhaus *et al.* 2001: 861). He offers as proof the frequent substitution of the subdominant by a Neapolitan sixth or by a «double dominant» (dominant of the dominant). Actually, in the functional system the Neapolitan sixth is just a variant of the own subdominant with a minor sixth, as we will see below, and is included in this function. On the other hand, it is perfectly possible to use a subdominant followed by a dominant of the dominant, which in this case does not substitute to the previous chord.

- The fifth may be perfect, augmented (<) or diminished (>). It does not usually miss, but its absence does not affect to the function.
- The seventh will be always minor, and may be present or may be omitted, although the former is the usual case.
- The 9th may be major or minor. When it is present usually the fundamental is absent (and vice versa), although there are examples of dominant chords with both the fundamental and the 9th.[36]

The use of these five sounds is open to three possibilities: its presence or its absence, the use of its natural or its altered version and its position (root position or inversions). This is the cause of such a number of variants that can represent the dominant function. It would be possible to establish a line to classify the chords according to their functional capacity, which could be defined as the degree of conviction of a chord to represent its function. In this sense, from the chords displayed in Fig. 7, the most convincing one [$\mathbf{D}^7$] would be at an extreme, while the functionally weakest [$\mathbf{D}_{5>}^{7\,9>}$] (which almost coincides with a variant of the subdominant function, except for the use of the leading tone, impossible in this last function), would be in the other extreme. The remaining chords would occupy the intermediate places according to the aforementioned possibilities: chords with fundamental are stronger than those without it, just like it happens for chords with natural sounds and those with altered sounds, and, of course, the root position is stronger than the inversions. Fig. 7 shows the original 5-note chord and several variants of the dominant function.[37]

Fig. 7 Origin and some variants of chords with dominant function

The observation of this figure allows us to answer some of the pending questions. One of them is the denomination of the diminished chord on the VII degree. This chord is considered here as a $\mathbf{D}^7$ without fundamental [$\mathbf{D}^7$] and, therefore, incomplete, weaker, although its destiny remains the tonic and its function that of dominant.[38]

There are two other chords whose denomination in the traditional harmony shows their weaknesses and their inconsistencies: the seventh chord on the leading tone and the chord of diminished seventh. First, both are chords built on the leading tone (in the absence of the fundamental), and the difference between them, therefore, is the type of

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[36] The famous «Rheingold! Rheingold!», from 1st Scene of *Das Rheingold* of Richard Wagner.

[37] Riemann introduced in his text the symbol < besides a number to denote an altered note by a half-tone higher, and the symbol > to denote an altered note by a half-tone lower. Of course, is also possible to use the real alteration (sharp, natural, flat) employed in each case besides the number.

[38] This chord was studied in some books as 'the diminished fifth chord', with certain pejorative nuance (perhaps due to its doubtful *status* and to its intervallic problems), when it is much easier understand it as a $\mathbf{D}^7$ chord without fundamental.

seventh, minor in the first case and diminished in the second, measured from the leading tone.[39] Second, this intervallic remark makes evident one of the major failings of traditional ciphering, namely the denomination of some chords from their characteristic intervals: diminished fifth chord, seventh chords on leading tone and diminished seventh, augmented sixth chords (which we will see later), etc., while the remaining chords are named from the corresponding degree (I, II, V, ...), all of which is a manifest incoherence of the system.

In the harmonic-functional system, the two mentioned seventh chords are considered as dominant chords without fundamental, with major ninth in one case $[\mathbf{D}_3^{7\,9}]$ and with minor ninth in the second case $[\mathbf{D}_3^{7\,9>}]$.[40] It can be proved that the chord sounds keep their numbering in the presence or absence of the root, as has already been said.

The dominant function has had a special meaning in tonal music, like a tension-charged chord that generates an unmistakable expectation towards its resolution in the tonic. Baroque musicians already discovered the emotive power of lengthening the duration of this dominant to increase the degree of satisfaction when the expected resolution is produced, and thus was born the so-called «organ point» (*Orgelpunkt*) or dominant pedal, a procedure frequent in the endings of many genres (prelude, fugue), or in the moments before the return of the main section (rondo) or of the recapitulation (sonata). Riemann studied this phenomenon in the final chapter of his *Vereinfachte Harmonielehre*, and explains it as an expansion of the cadential formula $\mathbf{D}_4^{6\,5}{}_3$ (Riemann 1893: 207-210), without forgetting that very often, the dominant pedal is followed by a pedal (almost always shorter) on the tonic. In both cases, the note held in the bass (or figured in some way) can be consonant or dissonant with the chords that sound above. This phenomenon is largely analogous to the «maintained dominant» that we will study in paragraph 3.8.2.

3.5.2. The dominant of dominant function[41]

The dominant of the dominant ($^{\mathbf{D}}\mathbf{D}$), also called in some texts «double dominant» (Dahlhaus) or *Wechseldominante* («change dominant», according to de la Motte),

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[39] Piston says it another way: "These chords are both dominant ninths without root and they both resolve regularly to the tonic triad, but are widely different in several respects" (Piston 1941/1959: 189).

[40] La Motte uses a special ciphering for these chords (1976/1989: 287-288), cancelling that of Maler, who nevertheless considered them as incomplete dominant chords with 9th, as we do here. The argument is that in the times of the Baroque and Classicism these chords were conceived as seventh chords from the leading tone, something which, perhaps being true, does not have to prevent us from interpreting them according to the methodology of the current harmony and unifying them within the large family of the dominant and its associated functions, without forgetting definitely the peculiarities of each of its variants. In the same way, methods of Schenkerian analysis are applied to thousands of tonal works that were not conceived thinking of *Ursatz*, as well as *Set theory* is used to better understand the music of Varèse or Webern, who never used that theory to compose.

[41] Riemann introduced the $^{\mathbf{D}}\mathbf{D}$ symbol for the dominant of the dominant, and the $^{\mathbf{S}}\mathbf{S}$ symbol for the subdominant of the subdominant, although with a somewhat different graphism to the current one. As he explains, "it is easier to represent the chord that includes the leading tone of the dominant [e.g. the F sharp in C major] as a simple chord a fifth away from the chord on the V degree [the **D** in C major], that is, as dominant of the dominant" (Riemann 1893: 110). Although the $^{\mathbf{D}}\mathbf{D}$ chord is used very frequently in both major and minor mode, the $^{\mathbf{S}}\mathbf{S}$ is associated just with the minor mode; it appears rarely, and its ciphering has been replaced by another closer to its origin within the given mode (**dP**).

cannot be considered a basic function, but the construction of its chords forming an extended family equivalent to that of the **D** (although in this case built on the II degree of the scale) and its destiny in the dominant one, make the **ᴰD** a function strongly rooted in the cadential process, within the group of predominant functions, which allows to draw in many cases a cycle of fifths that goes from the II degree to the V and the tonic (**ᴰD** - **D** - **T/t**). Therefore, the conditions for this function not to lose its identity and the possibilities of combination of sounds are exactly the same as in the dominant function (see Fig. 8).

Fig. 8 Origin and some variants of chords with dominant of dominant function

The function of **ᴰD** is constructed, as has been said, on the II degree of the scale, both in major as in minor mode. In both cases, the third of the chord is major, which implies the use of an alteration ($\hat{4}_<$) which is not in the key signature, but that is the most used of the possible alterations. It is not necessary to indicate this alteration in the ciphering, since the use of two uppercase letters already denotes that it is a dominant of another dominant, both with major third. As for the conditions of the sounds, the only difference is that in the **ᴰD** function the augmented fifth is not used, only the perfect and the diminished one. Precisely the use of the diminished fifth provides a special group of chords in which this fifth is located in the bass, what implies an interval between the fifth and the third (which is then above) of an augmented sixth (A flat – F sharp, which appears in those chords with 5> in the bass in Fig. 8), that has a strong tendency to resolve by opening to an octave within the dominant function (**D** or $\mathbf{D}_4^6{}_3^5$). The traditional Harmony usually considers these four chords separately, in an individual chapter, without integrating them as what they are, variants of the **ᴰD** function, and even assigning to them exotic names like «German, French, Italian and Swiss augmented sixth» (!). It is another example in which the chords have a purely interval designation, ignoring the most important matter, that is, their harmonic function. In the harmonic-functional system, however, these chords are considered as variants within the **ᴰD** function, certainly very peculiar, but sharing with many other chords the same mission within the cadential process, namely to exercise the function of predominant chords that lead to the dominant.

3.5.3. The subdominant function

The use of this function has historically been the subject of controversies: apart from the triad chord **S/s** (which does not present any problem, but it cannot be considered as usual in the classical cadence), the chords that are actually used within this function (in C major / C minor: F - A [flat] - D and F - A [flat] - C - D) present confusions in the ciphering. Many authors consider them as a II chord in first inversion,

without or with minor seventh (II^6 and II^{6}_{5}), which poses two problems. The first is that the IV - V - I melodic sequence in the bass is omitted in the cadence, being as it is by far the most used in this context. The second trouble originates in the apparent contradiction that arises when ciphering the two chords as inversions of supposed chords in fundamental position (II and II^7) that are barely used, something that removes credibility to the that derivation. Always remembering that this is the other possible predominant function, being therefore its mission precede the dominant function (with or without an intermediate ^{D}D), it is much easier to clarify the origin of the chords.

C maj/min: $\mathbf{S^6/s^6}$ $\mathbf{S^{6}_{5}/s^{6}_{5}}$ $\mathbf{S/s}$ $\mathbf{s^{6>}}$ $\mathbf{S^{6}_{5}{}_{6}/s^{6}_{5}{}_{6}}$ $\mathbf{S^{6}_{3}{}_{6}/s^{6}_{3}{}_{6}}$

Fig. 9 Usual chords of the subdominant function

The essential premise in the harmonic-functional system consists of dissociating in this case the harmonic theory (the building of chords by thirds) of the real practice in the composition, at the time that the mentioned melodic movement IV - V - I remains as the most relevant cadential bass. For this purpose it is considered that the bass of the whole chord family with subdominant function is the IV degree (in both modes) and that the third (major or minor) is obligatory. From this basic interval several possibilities are contemplated (Fig. 9):

- Addition only of the sixth (perhaps the oldest): $\mathbf{S^6/s^6}$. This chord is not considered an inversion of the II degree, but the normal position of the chord.[42]
- Addition of the fifth and the sixth (the most used during the Baroque and Classicism): $\mathbf{S^{6}_{5}/s^{6}_{5}}$. For the same reason, this chord is not considered an inversion of the II7 chord, but its usual position.[43]
- Addition only of the fifth (infrequent, more typical of the nineteenth and twentieth centuries): $\mathbf{S/s}$. This chord coincides, of course, with the triad chord previously defined as one of the three basic functions of tonality.
- A variant of the $\mathbf{s^6}$, the $\mathbf{s^{6>}}$ that is, the minor subdominant without fifth and with the minor sixth (the $\hat{2}\flat$ of the key), which is much better known as the

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[42] The statistical analysis shows that throughout history the $\mathbf{S^6/s^6}$ chord has been used infinitely more than the II grade in its root position in the cadential context. The explanation is simple and it is related to its melodic origin: in C major / C minor, for example, the pitch D (sixth of $\mathbf{S^6/s^6}$) is tied over or repeated (converted into the fifth of the $\mathbf{D}$), to then descend to the octave of the tonic, thus making the most frequent melodic movement in any end of phrase, section or movement: $\hat{2} - \hat{2} - \hat{1}$ over $\mathbf{S^6/s^6} - \mathbf{D^7} - \mathbf{T/t.}$

[43] What has been said for the previous chord is valid for it in regard to its historical presence and the movement of voices (although here the possibilities increase). This is the chord that Rameau already advanced (1722: 64) as a chord with added sixth to the perfect major or minor subdominant chord (*Sixte ajoûtée à l'Accord Parfait*) in its «irregular cadence» (our plagal cadence $\mathbf{S^{6}_{5}/s^{6}_{5}} \rightarrow \mathbf{T/t}$), where the sixth is a dissonance that resolves in the third of the $\mathbf{T/t}$, while – from the theoretical point of view – he derives it from the II degree chord with seventh when it is functionally located as the predominant chord before a dominant in the authentic cadence.

«Neapolitan sixth», and which D. de la Motte labels as **sⁿ** (alluding to its presumed though not proven geographical origin). Again, it must be remembered that this chord was not born as an inversion of the ♭II (**sG**), but was used almost exclusively in the sixth position for almost two centuries.[44]

- The potential inversions of these chords (in cadential contexts) are constructed from the original positions of the chords.

3.6. The tonicization and the modulation

As we have already advanced in Section 2, in the lines dedicated to Schönberg, it is important to distinguish between the process of tonicization and that of modulation, a difference that is associated with the cadential function and the form, as Caplin has perfectly settled down.[45] The tonicization (also denominated *Zwischenkadenz* [intermediate cadence] in Riemann), implies a unique line of analysis, since it is analyzed from a single tonic, whereas in the modulation a second line of analysis will be opened at some critical point for the new tonic that governs a wide temporal space.

Both tonicalization and modulation involve the use of one or more chords that announce the new tonic, in order to introduce the required alterations with respect to the current key and perform the cadential progression with chord functions taken from the future tonic. Both these chords and the new tonic are considered as «secondary chords» and «secondary tonic» with respect to the current tonality if it is a tonicization – the temporary establishment of any tonal region of the scale (diatonic or altered) as a momentary tonic.[46] Riemann explains that, in tonicizations, the attained secondary tonic must be indicated with the letter(s) that indicate(s) its function within the prevailing tonality. To do so, he uses a parenthesis – which isolates inside the secondary dominant and perhaps the secondary **S** and/or **ᴰD** –, followed by the secondary tonic thus announced and achieved (Riemann 1893: 141).[47] In the example below, the chords in parentheses are the secondary chords and the function inserted in the square is the secondary tonic (in this case the II grade of the key):

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[44] One of the first examples of the use of ♭II (**sG**) in a cadential context is found in *Prelude op. 24 No. 20* in C minor of Chopin. In this case the harmony of the measure that closes the second sentence begins by the tonicization of the **sG** itself as a model for a small progression that then descends half tone to end in the tonic: (**D**) → **sG** / **D⁷** – **t**.

[45] "[There are] two kinds of change in tonal focus – *modulation* and *tonicization*. Both procedures create the perception of a new tonic harmony, but they differ on whether this new tonic seems to represent the focal point of a new key, displacing the previous tonic from our primary attention, or whether this tonic is perceived as a more localized tonal emphasis within a prevailing key. More technically, the difference between modulation and tonicization depends on whether or not this new key is associated with a cadential function […] we can speak of a modulation to a new key if the new tonic is confirmed by a cadential progression in a formal context that functions to end a thematic process. By contrast, a new tonic dissociated from cadential articulation is better understood to represent the tonicization of a region in the prevailing key. This difference between modulation and tonicization can, of course, be challenged on a number of theoretical grounds" (Caplin 1998: 140).

[46] In the case of the secondary dominant the terms «applied dominant, intermediate dominant or artificial dominant» are also employed (Schönberg 1954: 28).

[47] The ciphering system has remained the same to our days (v. de la Motte 1976/1989).

C major: **T Tp D T**   (**s⁶ D**) $\boxed{\textbf{Sp}}$

Of course, not always the secondary dominant and predominant chords appear before the secondary tonic, but sometimes they do afterwards, and then a stronger confirmation of the secondary tonic closes the process. That is why I think it is convenient to add an arrow to make clear to which region the secondary chords in parentheses are related. In the first case of the examples below the secondary tonic (III degree of F major) appears after the corresponding secondary chords, but in the second case the secondary tonic (VII degree of A minor) is already heard before, and is confirmed after the secondary chords; for this reason, the mentioned secondary tonic should be remembered retroactively as the first announcement of that secondary tonic, something that is clear with the insertion of a pair of arrows in both directions:
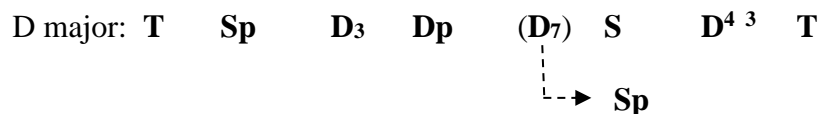

F major: **T**   **D₃ Tp** (= **s** $^{\textbf{D}}\textbf{D}_3^7$ **D**)→ $\boxed{\textbf{Dp}}$

A minor: **t**   **dP₃**   **tP**   **dP₃** ←($^{\textbf{D}}\textbf{D}_3^{7\,9>}$ **D**)→ $\boxed{\textbf{dP}}$

In the case of a secondary tonic heard before the secondary **D** (and **S** or $^{\textbf{D}}\textbf{D}$) but then not repeated, Riemann already proposed using a backward arrow to link these determinant chords with the tonic in a retroactive sense, as in the case of next example, where the secondary tonic (VI degree of C major) is heard before its determining chords (taken from Riemann 1893: 143):

C major: **T** **Tp** ← (**s₃** **D**) **T₃** **S** **D** **T**

[Original ciphering of Riemann: *T Tp ⌐(⁰S D⁺) T S D T*]

On the other hand, secondary chords do not always reach the announced goal, since composers, aware of the directional power of a secondary dominant, use this frustrated expectative as a form of unrealized implication that generates indubitable tension.[48] It is not easy to indicate in the analysis at the same time which *was* the announced tonic and what *is* the really achieved tonic. Riemann included the frustrated tonic in brackets, but at the same level as the other chords. My solution is to open a second level of analysis to indicate through a broken arrow which was the expected tonic, while the main line shows the chord that actually arrives in its place. This differentiation between the real harmonic level and the "virtual" level makes it possible to clearly express the different levels of harmonic discourse, thereby solving the problem of the implicated but not realized chords. In the following example, the secondary dominant clearly announces the **Sp** of D major, but instead the subdominant appears. Both things are clearly expressed by the following ciphering in two lines:

D major: **T** **Sp** **D₃** **Dp** (**D₇**) **S** **D⁴ ³** **T**

                                                    ⌐⊷ **Sp**

The display of two lines of analysis is also very useful to show the critical moment in which the main key of a sonata form movement is abandoned (momentarily) in favor of the subordinate key at the end of the transition, to give way to Secondary Group S, exposed in this last key, an example of modulation in the strict sense (since the new tonic is much more durable as it is associated at least with a complete thematic

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[48] Especially regarding the psychological mechanism of "implication-realization" and the concept of realization or frustration of expectative, as derived from the theory of information, see Meyer (1956) and Bent (1990), among others.

group).[49] In these cases it is convenient a double line of analysis that shows the functional filiation of the chords responsible for the modulation viewed from the home key (HK) and at the same time from the subordinate key (SK). Once the latter has been clearly established, the main key can be momentarily closed until its reappearance later. In the Fig. 10 – belonging to *Sonata No. 12 KV 332* of Mozart – the Transition begins with the tonicization of D minor, the **Tp** of the main key (F major), which is the **Sp** of the subordinate key (C major), to then go down through a progression that tonicizes in this case the **d** of the main key, which is the subordinate key itself in minor mode, closing that space with a half cadence in the dominant of this new key. Typically, the SK appears first in minor mode, changing to major mode with the arrival of the first theme of the subordinate thematic group (S_1), a frequent procedure at the end of transitions. The beginning of the Group S in this new key confirms the modulation process and allows closure for the moment of the domain of the main key. In this passage Mozart uses the so called German augmented sixth, which here is analyzed –as it was explained above – as a variant of the **^{D}D** function without the fundamental and including the seventh and the minor ninth (mm. 35-36).

Fig. 10 Mozart: *Sonata No. 12 KV 332 in F major*, I Mov. (mm. 21-42). End of Group P, Transition and beginning of Group S, with tonal double-line analysis

[49] The terminology used here for the sonata form comes from La Rue (/1970), Caplin (1998) and Hepokoski & Darcy (2006), revised in my doctoral thesis (Igoa 2014), where P is the first thematic group, T the transition, S the secondary or subordinate thematic group and K the conclusive zone.

The following example, taken from *Sonata No. 21 op. 53 "Waldstein"* by Beethoven (I movement, mm. 18-38), is a summary of several of the processes studied, which converge in the passage that includes the end of the first transition (T_1), the second transition (T_2) and the beginning of theme S_1. On the one hand, the double line of analysis shows the end of the main key and the arrival of the subordinate key, which here, surprisingly, is not the dominant (**D**), but the relative of the dominant, first apparently preserving its own minor mode (**Dp**), although later enhancing the third (**DP**), that is, E major,[50] which is reached through the tonicization of its subdominant (**s**) in m. 18-21, a ^{D}**D** and a long extended dominant.

On the other hand, the S_1 theme shows a secondary tonic (**Tp**) of E major followed by its secondary dominant, after which it does not return any more, but it falls into the **S** by means of a deceptive cadence, which makes it necessary to use the back arrow and dashed arrow to explain the frustrated harmonic process (Fig. 11):

Fig. 11 Beethoven: *Sonata No. 21 op. 53 in C major "Waldstein"*, I mov. (mm. 18-35). End of T_1, T_2 and beginning of Group S, with tonal double-line analysis

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[50] The explanation of the altered chords, their origin and their level of intermodal relationship can be found in paragraph 3.8.4. In this case, the traditional harmonic theory has never been able to explain satisfactorily the essence of this tonic in relation to the main one, since it is limited to ciphering it as a III degree of C major with change of mode from minor to major. However, the harmonic-functional system allows to see clearly that Beethoven – intuitively, of course – sought to go beyond the typical dominant (G major) as subordinate key, and thought about its relative (E minor), which is what announces at first the transition, but looking for a major mode for its S_1 theme he changed the mode when it reached it. The theoretical process that leads from C major to E minor and then to E major is found – in the opposite direction and as if it were his musical explanation – at the end of the exposition (mm. 74-86).

The Fig. 12 include, by way of summary, the basic and secondary functions, the inversions, the dominant function, the dominant of dominant function, the subdominant function and the nomenclature of the tonicizations (realized or implied).

Fig. 12 Summary table of chords in the harmonic-functional system

3.7. Enharmonic modulations

As for the modulations by harmonic means, it is possible to distinguish, firstly, the chromatic modulations, in which the existence of a mixed chord, common to the two involved regions, is not possible, since the sound or sounds of the first key belong only to it, and, once altered chromatically, only belong to the second key. The enharmonic modulations, however, do have a common chord, although in this case their denomination and sometimes their function are different in each of the keys implied, notwithstanding the sound is the same. Three are the most common types of enharmonic modulation that are used according to the chord on which they are based: diminished seventh, augmented fifth and augmented sixth.

The modulation based on the diminished seventh chord starts from the consideration that this chord can assume two functions ($\mathbf{D}^{\frac{9>}{7}}_{3}$ or $^{D}\mathbf{D}^{\frac{9>}{7}}_{3}$) in any major or minor departure key (A/a), as well as in the arrival key (B/b), which gives rise to the four types of modulation that are shown in Fig. 13.

$$A/a:\ \mathbf{D}^{\frac{9>}{7}}_{3} = \mathbf{D}^{\frac{9>}{7}}_{3}\ :\ B/b \qquad A/a:\ ^{D}\mathbf{D}^{\frac{9>}{7}}_{3} = {}^{D}\mathbf{D}^{\frac{9>}{7}}_{3}\ :\ B/b$$

$$A/a:\ \mathbf{D}^{\frac{9>}{7}}_{3} = {}^{D}\mathbf{D}^{\frac{9>}{7}}_{3}\ :\ B/b \qquad A/a:\ ^{D}\mathbf{D}^{\frac{9>}{7}}_{3} = \mathbf{D}^{\frac{9>}{7}}_{3}\ :\ B/b$$

Fig. 13 Types of enharmonic modulation based in the diminished seventh chord

In turn, each of the three possible chords of diminished seventh (G♯ - B - D - F * A - C - E♭ - G♭ * A♯ - C♯ - E - G; the remaining possible chords are transpositions of these chords with different names for the notes, but with the same sound) can be ascribed, according to its written denomination, to four keys, both in major as in minor form, as $\mathbf{D}^{\frac{9>}{7}}_{3}$, and as many others as $^{D}\mathbf{D}^{\frac{9>}{7}}_{3}$, providing in this way an enormous potential of combinations, which have been used from the Baroque to the 19th century.[51]

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The modulations based on the augmented fifth chord are much less frequent, and are almost circumscribed only to the 19th century, especially to the work of authors such as Liszt, Wagner, or Scriabin. Here, too, the chord can have several functional meanings, both in the departure key as in the arrival key. In major mode, exclusively, the chord can be $\mathbf{D}^{5<}$ (dominant with augmented fifth) or $\mathbf{sP}^{5<}$ (relative of minor subdominant with augmented fifth). In minor mode, exclusively, it can be $\mathbf{tP}^{5<}$ (relative of the tonic with augmented fifth), although in most contexts it can be better codified as

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[51] Works such as the *Chromatische Fantasie und Fuge BWV 903* or the *Fantasie und Fuge BWV 542* (just the Fantasie) by J.S. Bach, as well as the passage with which begins the development in the I movement of *Sonata nº 8 op. 13* of Beethoven, are excellent examples of this type of modulation.

$\mathbf{D_{6\,x\,5}}$ or $\mathbf{D^{6\,x\,5}}$ (dominant with sixth instead of fifth in any voice).[52] In turn, there are four different chords according to their written form (C - E - G♯ * D♭ - F - A * D - F♯ - A♯ * E♭ - G - B). As an example, the chord C - E - G♯ would have the following functions:

F major: $\mathbf{D^{5<}}$ E major: $\mathbf{sP^{5<}}$ A minor: $\mathbf{tP^{5<}}$ or $\mathbf{D_{6\,x\,5}}\,/\,\mathbf{D^{6\,x\,5}}$

Naturally, functional exchanges at the time of modulation are also possible here, which provides nine possibilities of connection (any of the three functions in the departure key may be transformed in any of the same three functions in the arrival key).[53]

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The modulation based on one of the variants of the augmented sixth chord ($^{D}\!\mathbf{\not{D}}^{9>}_{5>}$), the so-called German augmented sixth, has been, since classical times, one of the most employed, without losing its capacity for surprise, thanks to the unmistakable sound that characterizes it and the distance of the harmonic regions that it links. It consists of enharmonically transforming the $\mathbf{D^7}$ of the departure key into $^{D}\!\mathbf{\not{D}}^{9>}_{5>}$ of the arrival key. Generally it is used between major keys, or between one major and one minor key. It is also common to perform tonicizations on the $\mathbf{sG}$ (lowered II) by reversing the order of the chords. In any case, this modulation always links keys a minor second away, which implies a tonal distance of five alterations in the circle of fifths when the modes are equal. The basic types shown in Fig. 14, where the key A is always half tone above the key B (for example, C major and B major).[54]

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[52] The reason of this modal distinction lies in the voice leading to their resolution. In a C - E - G♯ chord with a $\mathbf{D^{5<}}$ function in F major, both the E and the G♯ must ascend to F and to A, what implies F major [Liszt: _Les Jeux d'Eau à la Villa d'Este_, mm. 49-52, where a $\mathbf{D^{5<}}$ chord alternates with the tonic in F♯ major]. In the same chord used as $\mathbf{sP^{5<}}$ linked to $\mathbf{T}$ in E major, the only voice movement goes from C to B, staying the E and G♯, respectively, as fundamental and third of a major tonic [Liszt, F.: _Vallée d'Obermann_, last measure, where this same chord is used as $\mathbf{sP^{5<}}$ before the final E major tonic]. Conversely, the same chord in the context of A minor has a clear dominant flavor, so that the only movement is now that of the leading tone G♯ going to A, since most of the time it functions as a $\mathbf{D}$ with a sixth instead of the fifth [Liszt: _Aux Cyprès de la Villa d'Este I_, mm. 1-33, where an extended $\mathbf{D_{6\,x\,5}}$ alternates with a $\mathbf{D_3}$, although finally goes back to the augmented fifth chord before the G minor tonic]. It is evident, however, that some of the resultant progressions (especially in the last two cases), may be analyzed also as one single chord (a tonic chord) with a 6 x 5 or a 7 x 8 _appoggiatura_, respectively.

[53] The passage between mm. 82-99 in the II movement of _Dante Symphonie_ of Liszt includes several augmented fifth chords which change of function during the modulation.

[54] Examples of this modulation can already be found in Haydn and Mozart. From this last composer we can remember the beginning of the development of his _Sonata KV 310_, which starts pointing to F major with a written $\mathbf{D^7}$ which is turned around enharmonically into a $^{D}\!\mathbf{\not{D}}^{9>}_{5>}$ in E minor. In his _String Quartet No. 19 V 465_, the mm. 115-116 shortly after the beginning of the development point clearly to B♭ major, but the $\mathbf{D^7}$ of this key is repeated in mm. 117-118 written as $^{D}\!\mathbf{\not{D}}^{9>}_{5>}$ of A minor. In the Sextet of _Don Giovanni_ (Act II, No. 19) the final intervention of Leporello ends in E♭ major, and its final $\mathbf{D^7}$ is transformed in $^{D}\!\mathbf{\not{D}}^{9>}_{5>}$ of D major for the entrance of Don Ottavio. Beethoven ends the section B₂ in the III movement of his _Symphony No. 9 op. 125_ in C♭ major, what compels to convert the $\mathbf{D^7}$ of that key in a $^{D}\!\mathbf{\not{D}}^{9>}_{5>}$ of B♭ major to come back to the main key (mm. 95-100).

$$\text{A: } \mathbf{D}^7 = {}^{\mathbf{D}}\!\mathbf{D}^{\overset{9>}{7}}_{5>} : \text{B [probable continuation: } \mathbf{D} \, (\mathbf{D}^{\frac{6}{4}\frac{5}{3}}) \, \mathbf{T}]$$

$$\text{B: } {}^{\mathbf{D}}\!\mathbf{D}^{\overset{9>}{7}}_{5>} = \mathbf{D}^7 : \text{A}$$

Fig. 14 Types of enharmonic modulation based in the augmented sixth chord

3.8. System extensions

Some additional observations will serve to complete and obtain the maximum of its possibilities from the system.

3.8.1. Parallel chords *versus* counter-chords

In Figs. 4 and 12 it is observed that there are some chords that have two possible denominations, one as a parallel and another as a counter-chord. The rule is to choose the first one, which is also the closest to the original function. There is a very clear exception, and it is the deceptive cadence in minor mode, in which it is more coherent to use the **tG** function for the VI degree (instead of **sP**), in order to explain the cadence as a not realized implication in which, however, the chord achieved partially maintains and shares the tonic function:

C minor: **D tG**

3.8.2. Implied tonality. Standing on the dominant

Some harmonic processes, especially in the space of development and analogous sections, often show the clear intention of culminating in a particular main or secondary tonic, and for this purpose, chords of this tonal region are used, especially functions of predominant and dominant type, although at the end the announced tonic is not attained in a clear root position, deviating the harmonic course towards another key. This process can bind several secondary keys and extend for a time, then characterized by great instability. The most visual way of representing this implied but not realized tonic is to insert it within brackets.

[T] [Sp]   Main or secondary tonic not reached, although the remaining chords of the passage clearly belong to this key (the most common is the use of some predominant function and standing on the dominant process)

In this class of processes and in many others it is frequent the use of a type of harmony that seems to be blocked over the dominant (with a greater or lesser presence of some real pedal note or figured pedal and alternation of the root position [$\mathbf{D}^{\overset{5}{3}}$] with $\mathbf{D}^{\overset{6}{4}}$, apart from other chords more or less consonant with the root note of that dominant). It is a thematic and harmonic process typical of the end of many baroque preludes and fugues, the end of the transition and especially the end of development in the sonata form, where its goal is to create an extensive expectative on the main key and

the first P theme that are coming back. It is what Caplin calls 'standing on the dominant', which can be abbreviated as maintained dominant in the proposed symbol.

DM   Standing on the dominant or maintained dominant

3.8.3. Modal interchanges

We have already seen that the three basic functions can be in major mode (**T**, **S**, **D**) or minor (**t**, **s**, **d**), depending on the mode of the key to which they belong, although modal exchanges are frequent already among the own basic functions (the functional dominant of the minor mode is the **D** taken from the major, and in this mode the use of the subdominant of the minor mode is very usual, for example). In the harmonic-functional system, the analysis of a passage in which the tonic goes through a limited space from a major mode to a minor mode or vice versa does not require more than changing the letter **T** for **t** or vice versa, and the same occurs with the cyphering of the other two basic functions.

3.8.4. Levels of intermodal relationship

The derivative or secondary chords, on the other hand, show in their initial format an opposite mode to that of the basic function of origin, that is, both the parallel and the counter-chord of a basic function in major mode are minor chords, and vice versa. Hence, the symbol always contains a capital letter and then a lowercase letter or the other way around (**Tp**, **sG**, etc.). However, as with basic functions, modal interchanges between homonymous keys (i.e., C major and C minor), changes of mode of single chords, interpolations and chromatic inflections in chords – as a consequence of the virtually unlimited use of alterations of an extended scale – are also common among secondary chords since the 18th century, and much more from the 19th century onwards.

Riemann asserted in his moment the possibility of integrating any own or altered chord within the framework of a single key, without losing the identity of that key: "It is about developing the feeling of tonality in its highest degree and also to make understand the most distant chord regions from the prevailing tonality" (Riemann 1893: 141). It is also what Mark DeVoto – in his extension of the *Harmony* of W. Piston (1941/1991: 447ff.) – called "extension of tonality or common practice", and Schönberg "extended tonality" (1954: 87ff.). This last author writes about this: "Remote transformations and successions of harmonies were understood as remaining within tonality [...] They function chiefly as enrichments of the harmony and, accordingly, often appear in a very small space" (Schönberg 1954: 76-77). And then he adds: "Enriched harmony makes for variety, especially when repetitions threaten to produce monotony" (1954: 84).

Dieter de la Motte has devised a system for ordering the gradual distancing that occurs with the use of such procedures (1976/1989: 155). However, both the terminology used and the explanation of the process are quite confusing, so I prefer to offer here my review and adaptation of what we can call 'levels of intermodal relationship', which are collected in Fig. 15 for two homonymous keys, C major and C minor. In all cases, the tonic function maintains its mode, in spite of the alterations

observed in the derived chords.[55] In this way, in the key of C major (left column), the first available chords are the diatonic chords derived from the three basic functions (Level I); the next available chords are those provided by modal interchange with C minor, that is, the borrowing of the derived chords from Level I of C minor to C major (Level II); then come the diatonic derived chords of the Level I of C major changed to its opposite mode, in this case from minor to major (Level III); and finally the chords provided by modal interchange with the Level III of C minor, that is, the borrowing of the derived chords from Level III of C minor to C major (Level IV). All this is possible without losing the dominance of the tonic in major mode as a reference. The same –but in opposite way concerning to the resultant modes – can be said of the column on the right (C minor), where the chords of the four levels are referred to the tonic in minor mode. Modal interchange and change of mode operations are performed with the derived chords as parallel and counter-chords of the three basic functions. It is very important to realize that the establishment of these four levels allows calibrate with accuracy the degree of proximity or distance to the basic diatonic level of the chords used within the tonality, with a range that goes from the closest proximity (the own diatonic chords) to the most remote chords.

- **Level I**. Diatonic chords derived from the basic functions of tonality. They have two pitches in common with the basic function, that is, the maximum closeness between triad chords. This is the original format that results from the normal derivation of parallels and counter-chords with respect to its basic function, explained above and illustrated in Figs. 4 and 12.
- **Level II**. Modal interchange: the chords derived from the basic functions in the major key (Level I) are used in the minor key becoming the Level II of this key, and the derived chords of the basic functions in the minor key (Level I) are used in the major key becoming the Level II of this key. There is just one pitch in common between the borrowed chords and the basic function.[56]
- **Level III**. Change of mode of the derived chords obtained in Level I. There is just one pitch in common between the changed chords and the basic function. The minor chords derived from each basic function in the major key (Level I) change their mode to major in Level III. The major chords derived from each basic function in the minor key (Level I) change their mode to minor in Level III. This implies in both cases the presence in the ciphering, for the first time, of two uppercase letters or two lowercase letters, since the parallel and the counter-chords have been changed of mode.[57]

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[55] The dominant and subdominant functions are more likely to change their mode. In minor mode, the functional dominant must be in major mode, and rather frequent is also the use of the major subdominant. Conversely, in major mode the subdominant is sometimes changed to minor mode, although this is not possible with the dominant function, for obvious reasons (in tonal music; modal music is another thing).

[56] In major mode, these are the chords usually known as 'lowered VI or lowered III degree'.

[57] In C major, for example, the own derived chords **Tp** and **Tg** change to major mode and become **TP** (minor parallel of the major tonic turned to major, that is, A major) and **TG** (minor counter-chord of the major tonic turned to minor, that is, E major), and hence the arrow that joins the Level I with the Level III, and in minor mode the same happens but in opposite modes. This Level III allows to properly explain the origin of the subordinate key used by Beethoven in the I movement of his *Sonata No. 21*, which may be understood therefore like a Level III where the **Dp** of C major is changed of mode to **DP** (see Note 50). The same – but in opposite modes – can be said of the leitmotiv associated with the helmet in the 3rd Scene of Wagner's *Das Rheingold*, where the tonic G♯ minor alternates with E minor, that is, the **sp** (the major parallel of the minor subdominant turned to minor), which later becomes the **s** (minor

- **Level IV**. Modal interchange: the derived chords changed of mode in the major key in Level III are borrowed by the minor key as Level IV in this key, and the derived chords changed of mode in the minor key in Level III are borrowed by the major key as Level IV of this key. In this last level there are no pitches in common, as an evidence of the maximum distance between the obtained chords and the basic function.

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subdominant) of the new tonic B major, which is itself the **tP** of G♯ minor, in a beautiful sample of intermodal relationships. The same progression – but referred to C minor alternating with A♭ minor – is used by R. Strauss to begin the first song of his *Vier letzte Lieder*, written almost one century later.

Functional-harmonic system
Levels of intermodal relationship

Fig. 15 Levels of intermodal relationship: tonic, subdominant and dominant function
Revision of terminology and elaboration of scheme: E. Igoa
[from *Harmonielehre* by D. de la Motte (1976/1989: 155)]

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